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THE KOENIG FORCE IN A COMPRESSIBLE FLUID

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In publications referring to the Koenig force (see, for example [1-3]), it is assumed that the acoustic wavelength is much larger than the separation between the dispersed particles. Such an assumption allows fluid compressibility to be neglected, but it is valid only for low frequency waves. On the other hand, in practice, for instance in ultrasound technology, radiation of quite high frequency (10^4-10^9 Hz [4]) must be considered. The wavelength of such radiation can be comparable to or even smaller than the separation between particles while remaining many times larger than their dimension. Obviously the neglect of fluid compressibility is then unjustified. The question arises: how does the structure of the Koenig force change when fluid compressibility is taken into account? This paper gives an answer to the question.

Thus we need to compute the force of radiative interaction (the Koenig force) of two rigid spherical particles whose centers execute small oscillations of circular frequency ω when the separation ℓ between the particles is comparable to the acoustic wavelength $\lambda = 2\pi c\omega^{-1}$. The speed of sound in the fluid is c, and the particles have radii R_1 and R_2.

We examine the issue of small parameters. First, we assume that two standard conditions are satisfied: the fluid vibration is potential, that is, $\mathbf{v} = \nabla\phi$ (ϕ is the potential of the fluid velocity $\mathbf{v}$); and $|\mathbf{v}|/c \ll 1$. The latter condition is indicative of the small amplitude of the wave field. Second, in the solution to the analogous problem for an incompressible fluid, two other small parameters are used: $kR_{1,2} \ll 1$ and $k\ell \ll 1$ ($k = \omega/c$ is the wavenumber), with $kR_{1,2} \ll k\ell$. Their small magnitude and the relation between them follows from the assumption $R_{1,2} \ll \ell \ll \lambda$. Relaxing the requirement $\ell \ll \lambda$ means that only one small parameter, $kR_{1,2}$, remains in which to carry out all expansions.

It is well known that radiation forces, including the Koenig force, are quadratic in the field. Considering this, the problem can be formulated thus: we must find the leading

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terms in the expansion of the Koenig force in the parameter $kR_{1,2}$, in the quadratic field approximation for arbitrary dependences λ and ℓ.

The radiation force $\mathbf{F}_j$ acting on the j-th particle (j = 1, 2) will be computed by the formula obtained in [3]:

$$\mathbf{F}_j = \rho_0 \left\langle \int\limits_{s_j} [\mathbf{n}_j (v^2 - k^2\varphi^2)/2 - \mathbf{v}(\mathbf{v}\cdot\mathbf{n}_j)]\, ds_j \right\rangle. \tag{1}$$

Here ρ_0 is the density of the undisturbed fluid, ds_j is the surface element of the j-th particle at rest, and $\mathbf{n}_j$ is the unit outward normal vector to this surface. The angular brackets denote time averaging. Since all terms in the integrand of (1) are quadratic in the field, ϕ and $\mathbf{v}$ can be reasonably calculated from an approximation that is linear in the field. Consequently, we can use the linearized equations of fluid motion and can restrict satisfaction of the boundary conditions for $\mathbf{v}$ at the surface of the particle at rest. Thus we must solve the boundary value problem

$$\Delta\varphi + k^2\varphi = 0; \tag{2}$$

$$\mathbf{n}_j\cdot\mathbf{v} = \mathbf{n}_j\cdot\mathbf{w}_j \text{ for } \boldsymbol{\rho}_j = \mathbf{n}_j R_j,\ j = 1,\ 2, \tag{3}$$

where $\boldsymbol{\rho}_j = \mathbf{r} - \mathbf{r}_j$; $\mathbf{r}$ is the radius vector of a point in the fluid; $\mathbf{r}_j$ is the radius vector of the equilibrium position of the center of the j-th particle; $\mathbf{w}_j = \mathrm{Re}\{\mathbf{U}_j \exp(-i\omega t)\}$ is the oscillation rate of the j-th particle; and $\mathbf{U}_j$ is the complex amplitude. We write ϕ in the form of a sum of two dipole potentials: $\phi = \phi_1 + \phi_2$. Here

$$\varphi_j = \mathrm{Re}\, \{a_{j\alpha} n_{j\alpha} h_1^{(1)}(k\rho_j) \exp(-i\omega t)\}\ (\rho_j = |\boldsymbol{\rho}_j|); \tag{4}$$

$h^{(1)}{}_1(k\rho_j)$ is the spherical Hankel function; a summation is carried out over the index α. It is evident that ϕ satisfies (2). Correspondingly, for the fluid velocity $\mathbf{v}$ we obtain $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$, where $\mathbf{v}_j = \nabla\cdot\phi_j$. We find the unknown coefficients $a_{j\alpha}$ from the boundary conditions (3) to an accuracy of the leading terms in $kR_{1,2}$: $a_{j\alpha} = -ik^2R^3{}_jU_{j\alpha}/2$.

Let us now switch to the calculation of the radiation forces. Considering the symmetry of the problem, it is sufficient to find the force $\mathbf{F}_1$ acting on the first particle. The force $\mathbf{F}_2$ is then easily found by reversing the notation for the first and second particles in the expression for $\mathbf{F}_1$. Setting j = 1 in (1) and substituting the expressions for ϕ and $\mathbf{v}$ into it, we have

$$\begin{aligned}\mathbf{F}_1 = {} & \rho_0 \left\langle \int\limits_{s_1} [\mathbf{n}_1 (v_2^2 - k^2\varphi_2^2)/2 - \mathbf{v}_2(\mathbf{v}_2\cdot\mathbf{n}_1)]\, ds_1 \right\rangle + \\ & + \rho_0 \left\langle \int\limits_{s_1} [\mathbf{n}_1 (v_1^2 - k^2\varphi_1^2)/2 - \mathbf{v}_1(\mathbf{v}_1\cdot\mathbf{n}_1)]\, ds_1 \right\rangle + \\ & + \rho_0 \left\langle \int\limits_{s_1} [\mathbf{n}_1 (\mathbf{v}_1\cdot\mathbf{v}_2 - k^2\varphi_1\varphi_2) - \mathbf{v}_1(\mathbf{v}_2\cdot\mathbf{n}_1) - \mathbf{v}_2(\mathbf{v}_1\cdot\mathbf{n}_1)]\, ds_1 \right\rangle.\end{aligned} \tag{5}$$

The integrand in the first term of (5) has no singularity in the volume bounded by the surface s_1. By transforming the surface integral to a volume integral, it is easy to confirm that the first term is identically equal to zero. Substituting ϕ_1 and $\mathbf{v}_1$ from (4) into the second term, we verify that it is zero as well. Expanding ϕ_2 and $\mathbf{v}_2$ in a series about the point $\mathbf{r} = \mathbf{r}_1$:

$$\varphi_2 \approx \varphi_2(\mathbf{r}_1) + \boldsymbol{\rho}_1\nabla_1\cdot\varphi_2(\mathbf{r}_1); \tag{6}$$

$$\mathbf{v}_2 \approx \mathbf{v}_2(\mathbf{r}_1) + (\boldsymbol{\rho}_1\cdot\nabla_1)\mathbf{v}_2(\mathbf{r}_1). \tag{7}$$

We substitute (6) and (7) and also ϕ_1 and $\mathbf{v}_1$ into the third term in (5). Omitting the straightforward intermediate calculations, we write the final formula

$$\mathbf{F}_1 = -2\pi\rho_0 R_1^3 \langle(\mathbf{w}_1\cdot\nabla_1)\,\mathbf{v}_2(\mathbf{r}_1)\rangle. \tag{8}$$

Together with (4), formula (8) solves the problem we have posed. We introduce the final expressions which hold for both $\mathbf{F}_1$ and $\mathbf{F}_2$:

$$\mathbf{F}_j = \mathbf{F}_{j1} + \mathbf{F}_{j2} + \mathbf{F}_{j3} + \mathbf{F}_{j4},$$

where

$$\mathbf{F}_{j1} = B\,\mathrm{Im}\,\{\exp[-(-1)^j ikl]\,(\mathbf{m}\cdot\mathbf{U}_1^*)(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{m}\}\,(kl)^{-1}; \tag{9}$$

$$\mathbf{F}_{j2} = (-1)^j B\,\mathrm{Re}\,\{\exp[-(-1)^j ikl]\,[(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{U}_1^* + (\mathbf{m}\cdot\mathbf{U}_1^*)\,\mathbf{U}_2 + (\mathbf{U}_1^*\cdot\mathbf{U}_2)\,\mathbf{m} - 6\,(\mathbf{m}\cdot\mathbf{U}_1^*)\,(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{m}]\}\,(kl)^{-2}; \tag{10}$$

$$\mathbf{F}_{j3} = 3B\,\mathrm{Im}\,\{\exp[-(-1)^j ikl]\,[(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{U}_1^* + (\mathbf{m}\cdot\mathbf{U}_1^*)\,\mathbf{U}_2 + (\mathbf{U}_1^*\cdot\mathbf{U}_2)\,\mathbf{m} - 5\,(\mathbf{m}\cdot\mathbf{U}_1^*)\,(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{m}]\}\,(kl)^{-3}; \tag{11}$$

$$\mathbf{F}_{j4} = -3\,(-1)^j B\,\mathrm{Re}\,\{\exp[-(-1)^j ikl]\,[(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{U}_1^* + (\mathbf{m}\cdot\mathbf{U}_1^*)\,\mathbf{U}_2 + (\mathbf{U}_1^*\cdot\mathbf{U}_2)\,\mathbf{m} - 5\,(\mathbf{m}\cdot\mathbf{U}_1^*)\,(\mathbf{m}\cdot\mathbf{U}_2)\,\mathbf{m}]\}\,(kl)^{-4} \tag{12}$$

$$(B = \pi k^4 R_1^3 R_2^3 \rho_0/2,\ \mathbf{m} = \mathbf{l}/l,\ \mathbf{l} = \mathbf{r}_2 - \mathbf{r}_1).$$

Let us compare these formulas with those from previous works. In [1, 2], a formula for the Koenig force in an incompressible fluid was given under the condition that both particles oscillate along the line joining their centers. In [3], this formula was generalized to the case of arbitrary direction of particle oscillation. This case is significantly more complex. Taking fluid compressibility into account shows that the structure of the Koenig force is of an even more complex character. First, there arise long-range terms in (9)-(11) for the Koenig force, which, unlike the "classical" term (12) are inversely proportional not to ℓ^4, but to ℓ, ℓ^2 and ℓ^3, respectively. In the limit of an incompressible fluid ($k\ell \ll 1$), it is possible to be restricted to the last term of (12) alone, which coincides with the results obtained in [1-3]. Second, the Koenig force begins to depend on the reradiation phase $k\ell$. As a consequence of this, it can approach zero and change sign for fixed ℓ. Third, $\mathbf{F}_1 + \mathbf{F}_2 \neq 0$. This is related to the fact that in an incompressible fluid, part of the momentum of the system is carried away to infinity [5].

In conclusion, we note that the change in the structure of the Koenig force has, in some sense, a universal character. Similar changes are observed for the Berkness force [5, 6]. The same effect also occurs in problems of radiative interaction in an electromagnetic wave field of electric charges [7] and magnetic moments [8].

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